Math 10A
Midterm I; **Friday, 7/6/2018**
Time: 2:10 PM **Name:** _____
Instructor: Roy Zhao **Student ID:** _____

- **DO NOT OPEN THE MIDTERM UNTIL TOLD TO DO SO!**
- Do all problems as best as you can. The exam is 80 minutes long. You may not leave during the last 30 minutes of the exam.
- Use the provided sheets to write your solutions. You may use the back of each page for the remainder of your solutions; in such a case, put an arrow at the bottom of the page and indicate that the solution continues on the back page. **No extra sheets of paper can be submitted with this exam!**
- The exam is closed notes and book, which means: **no class notes, no review notes, no textbooks, and not other materials can be used during the exam.** You can only use your cheat sheet. The cheat sheet is one side of one regular 8×11 sheet, handwritten.
- **NO CALCULATORS ARE ALLOWED DURING THE EXAM!**
- Justify all your answers, include all intermediate steps and calculations, and box your answers.

1. (16 points) Calculate the following limits and derivatives.

   (a) (3 points) $\lim_{x \to 10} \dfrac{x^2 + 5}{x - 5} =$

   (b) (3 points) $\dfrac{d}{dx}(x^4 + x^2 + 1) =$

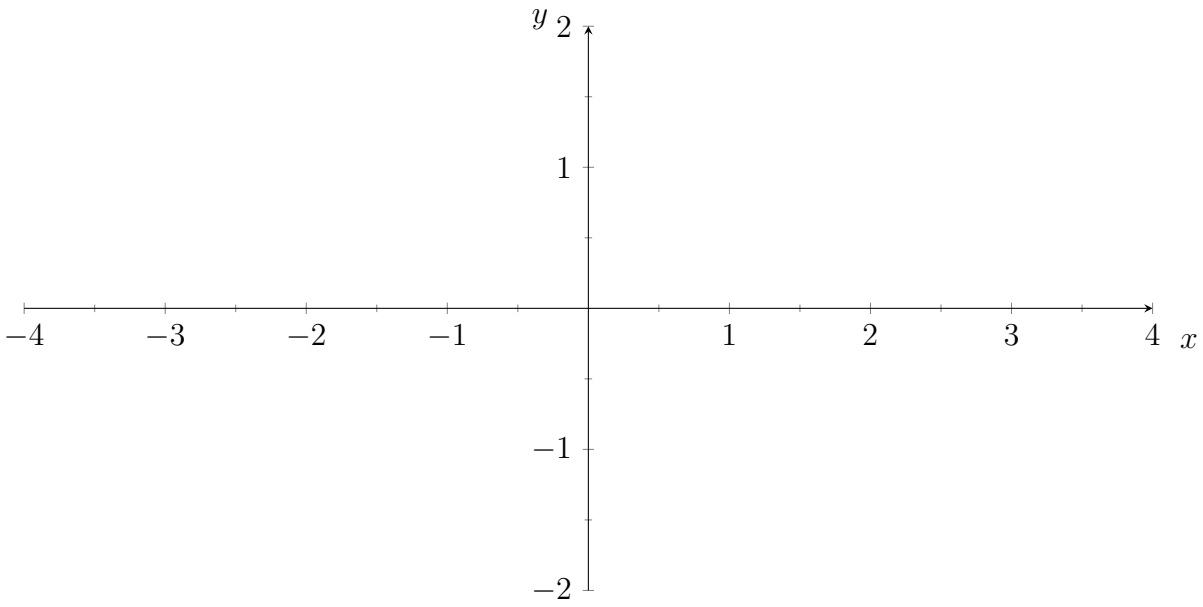
   (c) (5 points) $\dfrac{d}{dx}(\sin(e^x)) =$

   (d) (5 points) $\lim_{x \to 0} \dfrac{e^x - \cos(x) - x}{x^2} =$

2. (25 points) Let $f(x) = xe^{x+1}$.

   (a) (7 points) Calculate the derivatives $f'(x)$ and $f''(x)$ (factor if necessary) and find the zeros of $f(x), f'(x)$ and $f''(x)$.

   (b) (18 points) Sketch the graph of $f(x)$. The graph must clearly show: increase/decrease, concavity, asymptotes. Include your calculations to justify how you found these features. (Note: $2/e \approx 0.75$)



3. (15 points) A farmer wishes to divide his farmland along a straight river into 6 smaller identical rectangular plots by using one fence parallel to the river and 7 fences perpendicular to it. If he has 14 miles of fencing, what is the maximum area he can enclose?

4. (18 points) A conical cup that is $6cm$ wide at the top and $6cm$ tall is filled with water is punctured at the bottom and water is coming out at a rate of $\pi cm^3/s$. How fast is the height of the water changing when the height is $2cm$? (Note: The volume of a cone with radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$)

5. (16 points) (a) (8 points) Use a second order Taylor polynomial of e^x around $x = 0$ to approximate $e^1 = e$.

(b) (8 points) Use one iteration of Newton's method to find the critical point of $\sin(x) - x^2$ starting with a guess of $x_0 = 0$.

6. (10 points) Circle True or False. (1 point for correct answer, 0 if incorrect)

(a) True False If f is defined at $x = 0$, then $\lim_{x\to 0} f(x) = f(0)$.

(b) True False If the function f is not invertible, then there is no x such that $f(x) = 7$.

(c) True False The range of an invertible function f is the domain of the inverse f^{-1}

(d) True False If f is continuous on $[0, 2]$, then $\lim_{x\to 1} f(x) = f(1)$.

(e) True False If the derivative of a function f is negative at $x = c$, then $f(c) < 0$.

(f) True False If the derivative of f is increasing, then f is increasing as well.

(g) True False If $\lim_{x\to c} \frac{f'(x)}{g'(x)} = L$, then $\lim_{x\to c} \frac{f(x)}{g(x)} = L$.

(h) True False If $T(x)$ is the third order Taylor polynomial of $f(x)$ centered at $x = a$, then $f(a) = T(a)$.

(i) True False Newton's method always converges to the zero of a function.

(j) True False If we are using Newton's method to approximate $\sqrt{17}$ with an initial guess of $x_0 = 4$, then we apply Newton's method to the function $f(x) = \sqrt{x}$.